Stiffness and Deflection Testing on the LongMill

Introduction

This report was written by Johann, one of the engineers here at Sienci Labs. I am adding my own "translation" and comments to this report to add context to the testing and data presented.

Rigidity is one of the most important factor in terms of a hobby CNC's performance. A machine which is more rigid will typically be able to cut faster and more accurately. There are many factors that can affect the rigidity of a machine, such as with it's design, size, and setup, but also the way the tests can significantly affect the results.

At Sienci Labs, we want to offer the best performing machine possible at a reasonable price. We hope through this report we can provide confidence to our users that 1) the LongMill is a rigid and highly optimized machine and 2) users should expect exceptional performance in a hobby CNC context.

In this report, Johann considers both the predicted results and real life testing in the LongMill's rigidity. Also, Johann considers the real life scope of forces and use that a CNC machine can expect to experience to give data that is relatable for the general user.

Objective

1. Establish standardized testing procedures for cross-machine deflection testing*
2. Build a deflection model of the Longmill MK2 and identify the areas of greatest improvement

**At the current time, we've not been able to find standardized testing procedures or methods within our industry. This is why part of the goal was to establish some sort of standard procedure for testing any CNC machine, if we in the future wish to compare rigidity between different machines.*

Results

When using the LongMil

Testing of two LongMill MK2s show the following deflection figures.

Before testing began, all V-wheels were adjusted in pairs so that there is ~7.5N rolling resistance for the XZ gantry and each of the Y-axis gantry. The anti-backlash nuts are also adjusted to a quarter-half turn post-engagement. Any residue from the rails is also cleaned before the test is performed.

For the 10N run, the results are as follows:

(Tested at 10N)	**X Axis Deflection***	**Y Axis Deflection**
48x30 Longmill MK2	2.8 thou / 0.072mm	3.2 thou / 0.080mm
30x30 Longmill MK2	2.3 thou / 0.057mm	3.0 thou / 0.076mm
12x30 Longmill MK2	1.9 thou / 0.049mm	3.0 thou / 0.076mm

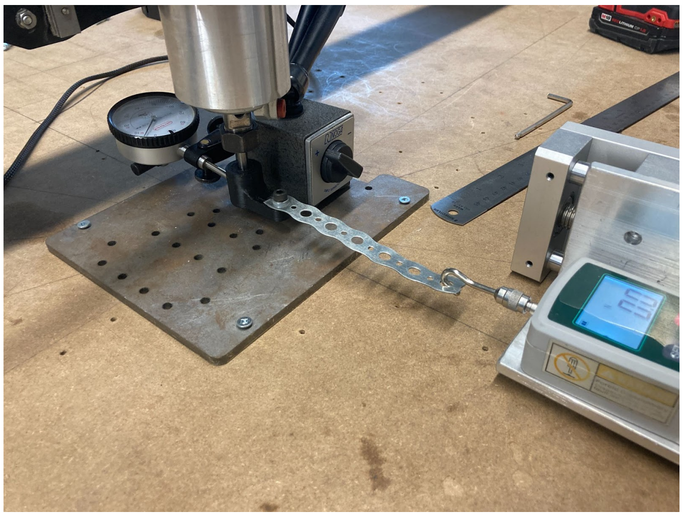
For the 25N run, the results are as follows.

(Tested at 25N)	**X Axis Deflection**	**Y Axis Deflection**
48x30 Longmill MK2	14.2 thou / 0.361mm	20.7 thou / 0.525mm
30x30 Longmill MK2	12.3 thou / 0.313mm	18.5 thou / 0.470mm
12x30 Longmill MK2	13.0 thou / 0.330mm	18.2 thou / 0.461mm

**To give some context to the numbers, 3 thou, or 0.1mm is roughly the thickness of a sheet of paper.*

Areas that are worth further exploring

4. While it's tempting to conclude that the 30" machine is considerably stiffer than the 48" machine by the virtue of its size, the difference measured between the 48" and the 30" is considerably larger than what is predicted using beam deflection models. There is some evidence to suggest that this variance comes from factors we did not control for during the test (e.g. wear and tear of the V-wheels & delrin nuts, the mounting rigidity of the wasteboard, variation in rigidity of the MGN rails, etc.). Case and point, the original XZ gantry on the 30" machine deflected more than even the 48" machine during initial testing until the gantry from the 48" was swapped in. To further narrow this down, we can consider:

    a. Identify where the variance is coming from by running further tests
    b. Directly control for V-wheel tightness (Some torque / tightness measuring tool)
    c. Measure the relationship between V-wheel wear / tightness and deflection
5. The current test setup has the dial indicator positioned opposite the force gauge with the 0.25” milling bit sandwiched in the middle. While this arrangement is good for deflection measurements in a single direction, it is not well suited for measuring the deflection envelope from both directions since the dial indicator and force gauge needs to be repositioned half-way through the test.

   The errors that this introduces should be below 1 thou, but there are a few ways this can be improved:
    a. Use a test indicator / hall effect sensor to improve clearance
    b. Load the bit at a 45 degree angle and decompose out deflection in the X and Y direction
6. Current measurements suggest that our 30” machine performs similarly to the Shapeoko 3 without the stiffer SO3 Z-axis upgrade (quad linear rail blocks). Which may hint at issues with measurements (https://community.carbide3d.com/t/backlash-deflection-and-vibration/28669)

Deflection Testing Procedures

To establish a standardized way to easily measure the rigidity of desktop CNC.

Test setup

Tools needed

Tool	Notes

0.001" Resolution dial indicator & (magnetic) stand	-
Flat steel plate	Secured to the waste board with work holding and acts as a base for the dial indicator stand
Force gauge / luggage scale	Holding the force gauge by hand is not recommended since it will be challenging to keep forces steady while reading the dial indicator. Instead we recommend building a simple jig to secure and move the force gauge (See the load application section)
0.25" End mill	-
3D printed parts	End mill clip and rope guide

Test procedure

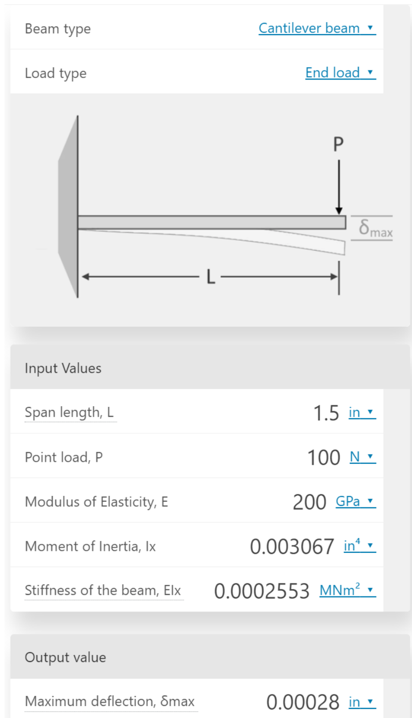
18. Jog the machine to the center of the wasteboard
19. Adjust the router's position in the router mount so that the collet is 0.75" off the wasteboard when the router mount is at its lowest position
20. Secure the 0.25" end mill / gauge pin in an inverted orientation with 1.25" stick out
21. Raise the end mill 1" off the wasteboard
22. Attach the end mill clip to the bit
23. Secure the steel plate to the wasteboard near the end mill using any work holding (T-track clamps, wood screws etc.)
24. Place and lock the dial indicator stand onto the steel plate
25. Decide on the axis to apply load and orient the cutout in the clip to face away from that direction
26. Position the dial indicator against the end mill exposed in the cutout
27. Apply a 10N/2.25lbs* load in the direction away from the dial indicator and note the reading
28. Remove the load and note the change in reading, this will be considered deflection in the positive direction
29. Being careful not to bump into the spindle or the machine, remove the dial indicator and rotate the spindle to the opposite side
30. Reposition the dial indicator so that it is up against the end mill exposed in the cutout
31. Note the reading on the dial indicator
32. Apply a 10N load in the opposing direction and note down the change in reading, this will be considered deflection in the negative direction
33. Add the positive and negative deflection readings together and divide by 2 to get deflection for the axis being measured
34. Repeat step 7-12 three times and take an average before moving on to measuring the other axis

***A load of 10N was chosen as it represents a fairly reasonable amount of load on an end mill as it is cutting. Additional data and recordings are covered in a later section.*

Notes on test design

0.25” Drill bit

An inverted milling bit is used as it is readily available in most CNC shops and the additional deflection it adds is an order of magnitude smaller than the typical deflection measurements.



That said, I did measure a runout of 4 thou when the milling bit is installed in the inverted direction. To prevent this runout from going into any measurements, we can do one of the following:

4. Prevent the bit from rotation during measurements
5. Measure against other parts of the machine (e.g. the collet)
6. Use a gauge pin

The first option is selected since it allows for easy interpretation of the measurements, and it is relatively easy to hold the bit in place while measuring

Cutting forces

From the purposes of deflection measurements, a 25N load is chosen as it is within an order of magnitude of typical cutting forces calculated from machine power and is high enough to result in clear measurements.

Assuming a 1.25 HP router with a 0.25” bit and an RPM of 30000, the [theoretical maximum cutting force]() is 21 pounds / 93N.

$$F_{t,max} = \frac{63025 \times P}{r_{tool} \times n}$$

https://community.carbide3d.com/t/carbide-compact-router-max-power-max-torque-and-torque-curve/23458

There is quite a bit of research trying to characterize and model cutting forces in wood, that said these models are usually highly complicated (4+ variables) and use settings/tools that are not common to the benchtop CNC market (>0.5” bits at 4000+mm/s). An example of 3 papers below.

	Bit Diameter	DoC	Feed & Speeds	Cutting Forces
Douglas Fir (Link)	40mm	0.5mm/1.5mm	5000mm/s 13867 RPM	~20N / 40N
MDF (Link)	20mm	2mm	1500-4000mm/s 15000RPM	~13N
Maple & Oak (Link - Worth a read)	20mm	30mm	2000mm/s 3000RPM	Forces normalized

Forum users usually suggest cutting force would be under 50lb for handheld router based CNCs and caveat that the actual load would be a lot lower (link, link, link). I wasn’t able to find anything more concrete or broken down by bits / feeds & speeds.

Instead of solely relying on empirical data, cutting forces are determined experimentally by running the following tests.

Note that:

4. Rolling resistance is ~5N and has been subtracted from the figures below
5. The figures below reflect the reading of the force gauge when cutting forces have stabilized
6. Figures are all rounded to the nearest 5N

Material	Bit	DoC	Feed & Speeds	Slotting	Conventional (0.5D step over)	Climb (0.5D step over
Maple	0.25” 2 Flute Upcut Bit	2mm	3500mm/min (Router Setting 3)	10N	10N	0N
		4mm	3500mm/min (Router Setting 3)	15N	15N	0N
		4mm	3500mm/min RPM (Setting 1)	Unstable reading	20N	Runaway scenario
Plywood		4mm	3500mm/min (Router Setting 3)	Unstable reading	Unstable reading	5N
		2mm	3500mm/min (Router Setting 3)	10N	5N	Runaway scenario
	0.125” 2 Flute Downcut Bit	8mm	3500mm/min (Router Setting 3)	15N	10N	0N
		6mm	3000mm/min (Router Setting 3)	10N	10N	-5N

Since we can only capture cutting force along the feed direction, we will assume that

3. Forces perpendicular to feed during climb milling will be similar to forces along the feed direction during conventional milling
4. Forces during slotting will be a combination between conventional and climb milling forces by vector addition

Since conventional / climb milling would be the most common operations performed as compared to slotting, **we can take the 10N measured as the force most typical to average cutting.**

One last note is that the reading only reflects cutting forces during regular cutting, and it is observed that cutting forces can be an order of magnitude higher when the router is close to stalling / when the bit starts to exhibit chatter.

To add some additional notes, there are a few limitations on how rigid a machine needs to be. At a certain point, the rigidity of the structure of the machine becomes less of a factor compared to external factors, such as the amount of deflection in the end mill, spindle power, and vibration. At a certain point, the deflection from the tool becomes a larger factor than the deflection coming from the machine itself. In essence, this means that there also diminishing returns on a machine's rigidity as well.
I bring this up to our favor, because we have competitors with similar machines that use more expensive parts. Although the price of a machine doesn't necessarily imply that machine is more rigid, there is a better chance that it'll be structurally stronger.

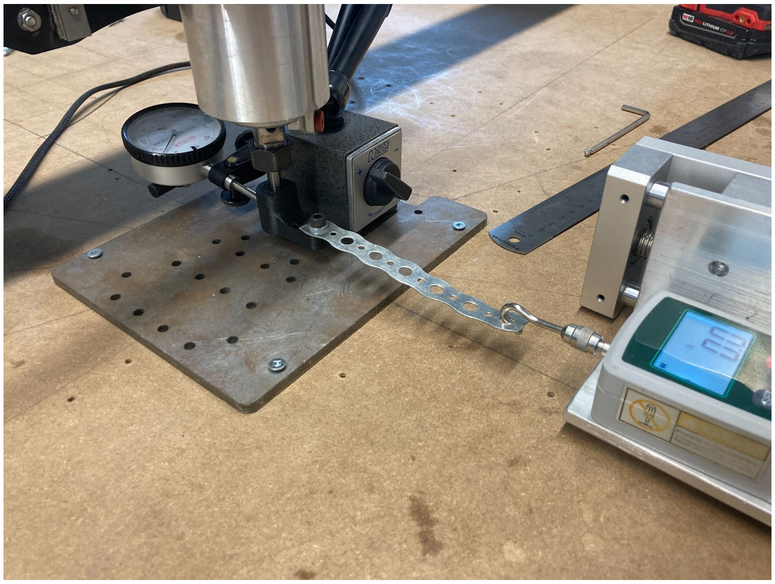
This might be important to some users who plan on using larger tooling or cut harder materials, but from my personal experience, the rigidity of the machine has become less of an important factor in terms of productivity of the machine because of the relatively small loads used with hobby CNC tooling and the Makita router.

Load application

Pulley system

A pulley system is quite convenient to use since it is not secured to the wasteboard and weights can be hung off the side of the wasteboard. That said, it is observed that friction losses can be quite significant so it's not recommended (e.g. ~15N error when using a quarter inch nylon rope, ~4N error when using a thinner string).

Direct loading to bit



The most effective way to apply load we have found so far is to use some pipe strapping that goes from the bit (with a clip) to the force gauge that is in turn secured to a table mounted vise.

Backlash considerations

Backlash is measured before the test to understand how much slop can be expected in the system.
https://machmotion.com/blog/what-is-backlash-and-how-to-correct-it/#:~:text=Backlash%20can%20be%20measured%20by,when%20an%20axis%20changes%20direction.

To reduce the impact of backlash in the final results, the machine is jogged 1mm towards and away from the direction in which load is applied.

Hysteresis

Repeated testing showed some hysteresis behavior in the system, where the dial indicator would fail to return to 0 after being loaded. This is a somewhat different phenomenon to backlash since it does not manifest when there is no load. The current test will attempt to capture hysteresis and backlash in overall deflection since all 3 effects will affect the final cut that users see.

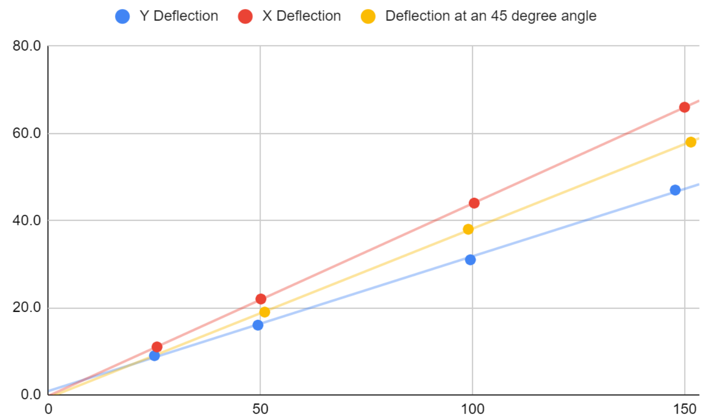
https://www.linearmotiontips.com/whats-the-difference-between-backlash-and-hysteresis-in-linear-systems/

Notes on stiffness rating calculations

The section describes the assumptions used in generating the stiffness rating and the rationale behind.

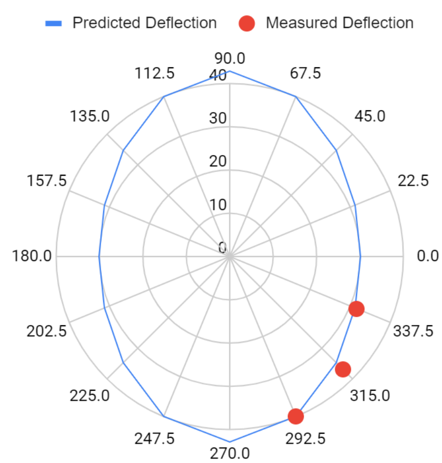
Linear relationship between load and deflection

This is almost a certainty even without testing. That said, I performed this test to make sure any measurements at 100N can be proportionally scaled up / down with minimum error which is indeed the case.



Vector addition in X & Y

To capture the stiffness on both the X and the Y axis in a single metric. It is important that deflection in the cardinal directions (once normalized to a given machine size) can be added together using vector addition and easily predict off axis deflection.



Testing shows some evidence of additional “looseness” off axis over what is predicted using measurements in the cardinal direction. However the error is quite small (6.3% / 2 thou @45°) and it can be observed that deflection off axis still tracks the deflection ellipse generated using cardinal measurements.

Compensating for axis length

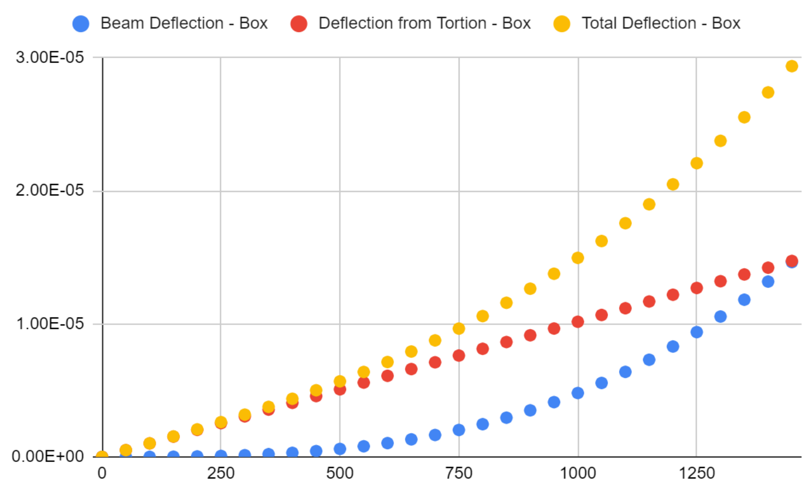
The idea is to take measurements at the center and the edges of the machine and adjust for axis length by extrapolation / interpolation.

$$\delta_y (L_x, L_y) = aL_x^m + bL_y^n + c$$

There is however the following complications:

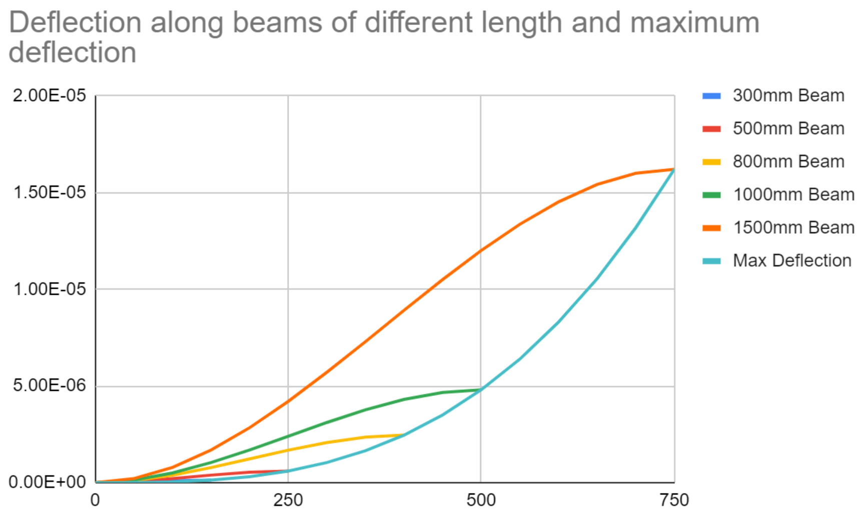
4. **Choosing the fixed powers n and m**
   There are multiple length dependent deflection modes that grow with different fixed powers. For example, deflection caused by the beam deformation of the Y-axis rail grows with L^3 while deflection caused by torsion grows linearly (under small angle approximations). On the X-axis, deflection caused by lead screw / belt stretch increases linearly with axis length.

   Depending on the machine being tested, the dominant power may be different and it may not be fair to assume the factor that is dominant.



5. **Practical challenge measuring at origin**
   At times it may be impractical to take measurements at the origin / edges of the machine given interference with the Y axis rails / limitations in table size. Taking measurement of a small offset (e.g. 50mm) from origin seems like a good option at first, however this is still problematic since deflection measurements made along a beam do not lie on the curve that describes maximum deflection for beams of varying length, so extrapolating

from such points will introduce additional errors.



6. **Dependencies between L_x & L_y**
   So far we have assumed that deflection changes along L_x & L_y are independent to one another, that said, there is some evidence to suggest that there is some dependency between the 2 variables. More specifically, deflection along L_x increased more at the center of the machine

Possible solutions / workarounds
Without actual machines to test with, further assumptions may inadvertently introduce significant errors into overall deflection figures.

As of the time of writing, we will only focus on overall deflection in aggregate without axis length normalization.

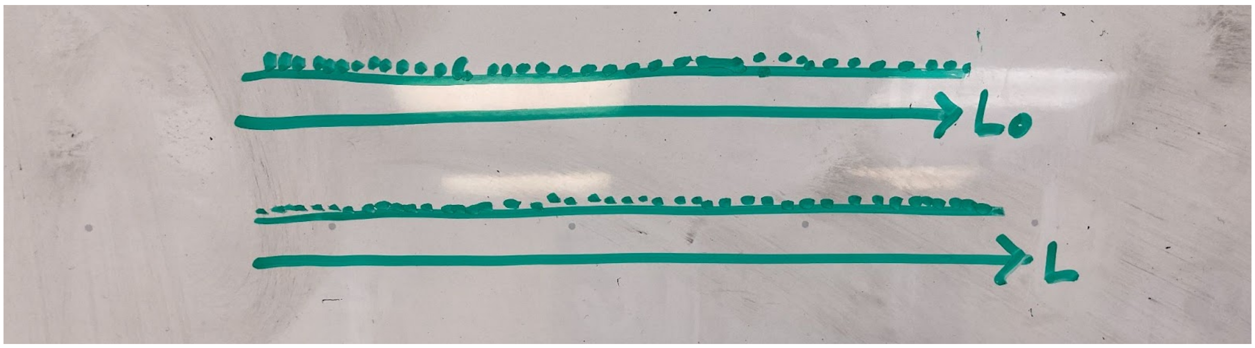
Belt deflection

Belt deflection doesn't apply to the LongMill because it doesn't use belts, but many hobby CNC machines do. Leadscrews are generally more accurate and don't stretch to the degree that belts do, so in essence can be more accurate. So even though a machine's structure might be rigid, the belts can have a large impact on the precision of a CNC machine. We have considered using belts in some applications, as they do offer some design and cost advantages, so this report helps us understand the implications of this type of system better.

Modeling

Intuition suggests that belts are less stiff the further they are away from the edge. Furthermore, since belts cannot resist compression, stiffness would depend on load direction once pre-tensioning is overcome.

To model and verify these behaviors, we first define a belt that is of length L_0, which due to pre-tensioning has been stretched to length L_0.

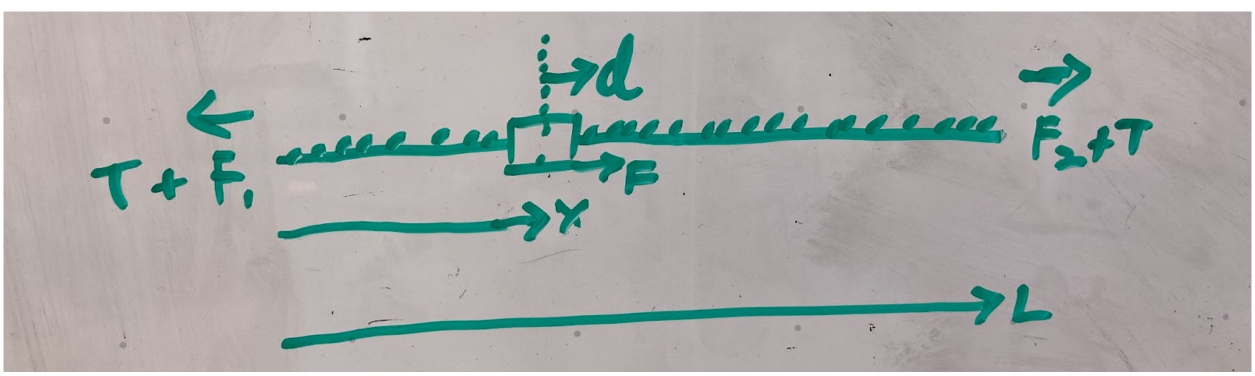


Assuming a pre-tensioning of T and a tensile modulus λ, the stretch can be characterized as follows:

$$T = \frac{L - L_0}{L_0}\lambda$$

Defining elongation using ε, the equation is simplified to:

$$T = \varepsilon\lambda$$



To model the behavior of the belt under load, a force F is applied at position x, which deflects the belt by the distance d. The changes in tension F_1, F_2 can be described as follows:

$F_1 = \frac{d}{x_0}\lambda$

$F_2 = \frac{-d}{L_0 - x_0}\lambda$

Expressing x_0 and L_0 in x and L gives:

$F_1 = \frac{d(1+\varepsilon)}{x}\lambda$

$F_2 = \frac{-d(1+\varepsilon)}{L-x}\lambda$

Where it can be observed that pre-tensioning “stiffens up” the effective spring constant by the factor $1 + \varepsilon$. Note that ε is usually a very small number compared to λ.

Expressing ε in terms of T and λ gives:

$F_1 = \frac{d(T+\lambda)}{x}$

$F_2 = \frac{-d(T+\lambda)}{L-x}$

Note that $F_1 + T$ and $F_2 + T$ cannot have a negative value so an effective “floor” of $-T$ exists for F_1 and F_2, at which point the belts are not longer in tension.

To summarize, F_1 and F_2 are piecewise functions that can be described as follows:

	When F_1 or F_2 is $\geq -T$	When F_1 or F_2 is $< -T$
$F_1 =$	$\frac{d(T+\lambda)}{x}$	$-T$
$F_2 =$	$\frac{-d(T+\lambda)}{L-x}$	$-T$

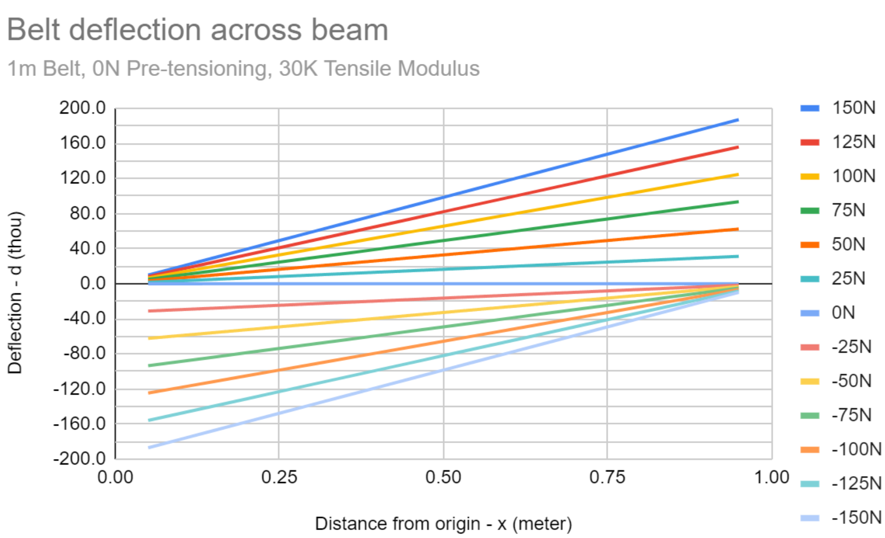
Consider equilibrium condition about x.

$F = (F_1 + T) - (F_2 + T) = F_1 - F_2$

F can thus be described with the following function

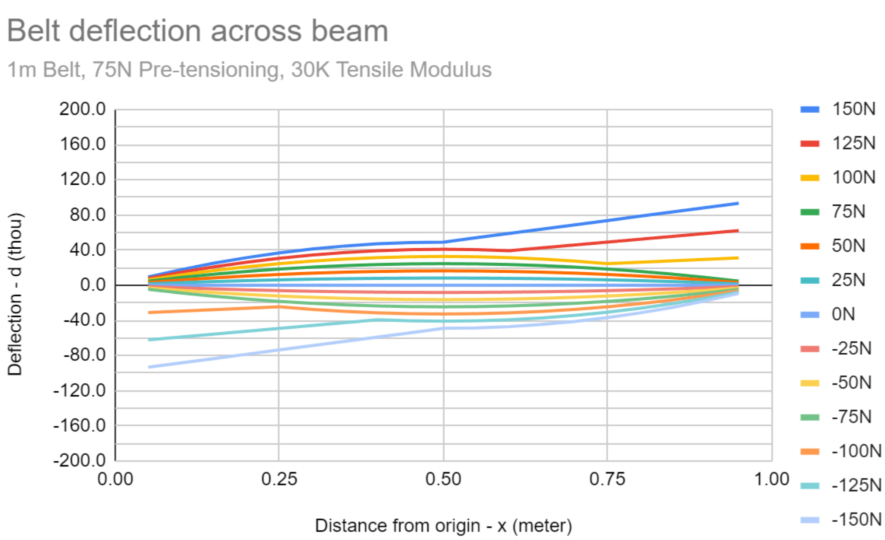
	When $F_1 < -T$	When $F_1\ AND\ F_2 \geq -T$	When $F_2 < -T$
$F\ =$	$-T + d(T + \lambda)\frac{1}{L-x}$	$d(T + \lambda)(\frac{1}{x} + \frac{1}{L-x})$	$d(T + \lambda)(\frac{1}{x}) + T$

Solving deflection d for various load and pre-tensioning values yield the following:



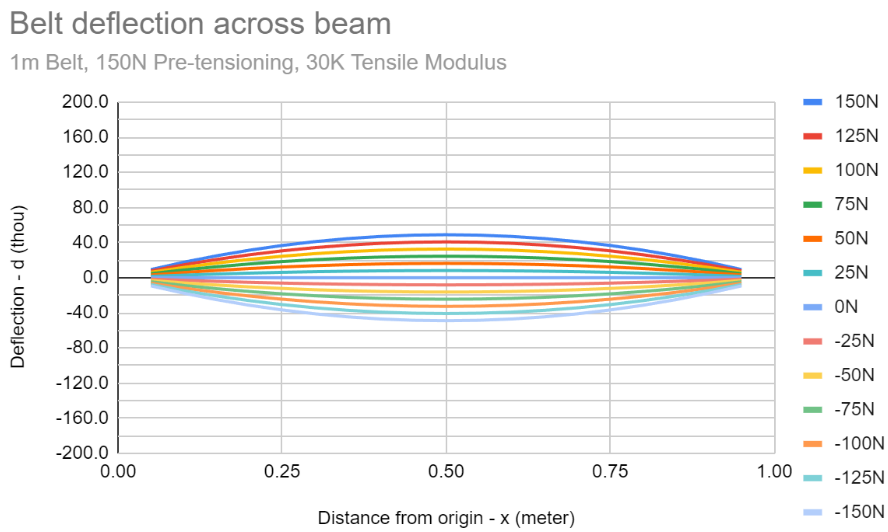
When $T = 0N$

- Belt deflection varies linearly across x and stiffness depends on load direction (i.e. if F is +ve or -ve).



When $T = 75N$

- Stiffness is independent of load direction for loads smaller than T across the entire beam
- Stiffness is still dependent on load direction for loads greater than T
- Stiffness is independent of load direction at the center of the beam for loads up to $2T$



When $T = 150N$

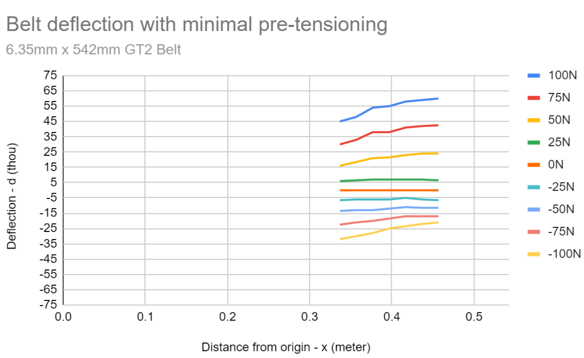
- As long as tensioned is maintained on both belts, further increases in T do not reduce deflection significantly. For example, increasing T from 75N to 150N only reduces deflection at the center of the beam by -0.093575%.

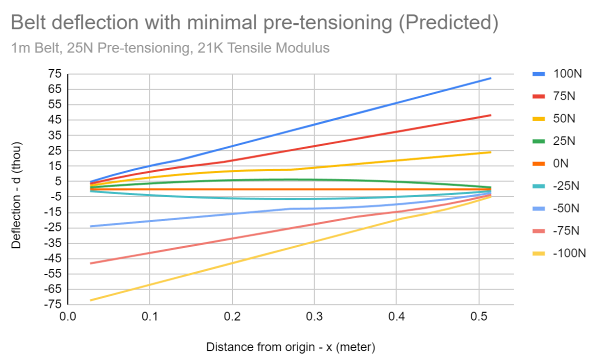
Validation

To double check the model's validity, an unbranded 542mm long, 6.35mm wide GT2 belt belt is tensioned and secured on two ends. Load is then applied along the belt and deflection measured.

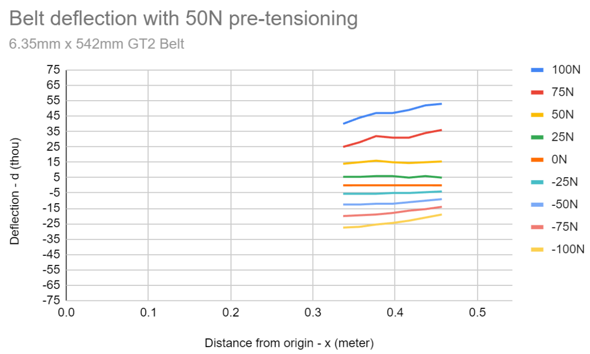
Note that the tensile modulus of the belt is measured to be around 21,000 N/mm/mm. This value is very close to values suggested for 6.35mm GT2 belts (reference 1, reference 2).

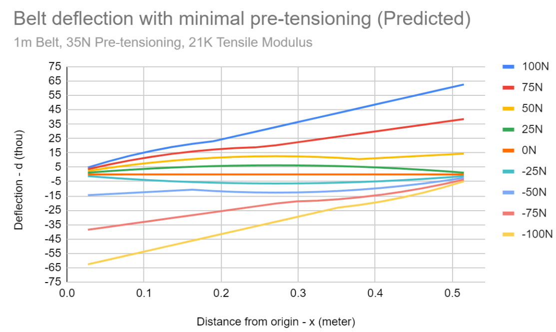
With minimal pre-tensioning, the belt exhibits increasing stiffness asymmetry the further away load is applied from the center. This corresponds quite well to the 25N tension belt model.



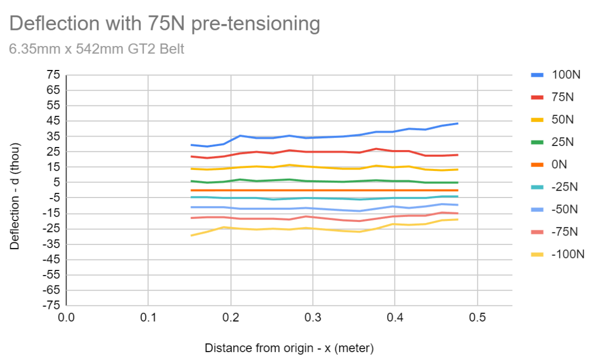


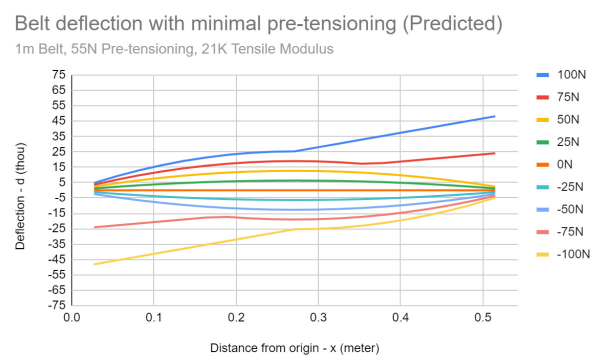
With 50N pre-tension, overall deflection is reduced and the 50N deflection line (dark yellow) remains flat across the length measured. Although these results do not correspond very well with the belt model at 50N, they match the model at 35N quite well. This is likely due to the way pre-tensioning is set, where a force gauge is used to pull on the belt through a pulley before the pulley is secured using grub screws, this means that friction at the pulley would reduce effective tension from the force gauge.





Results at 75N pre-tensioning shows a further reduction in deflection for the 100N deflection line (dark blue) , with the 75N deflection line (dark red) now trends downward as predicted. That said, additional divergence between the model and the results is observed even when the model adjusted to 55N pre-tensioning.





As things stand, evidence suggests the model is likely to be accurate in broad strokes. However, additional testing should be done to reconcile the errors observed before it is used to normalize deflection against axis length.

Implications for deflection testing

To quantify and isolate the effects of belt deflections, the variables T and λ should ideally be measured. This will allow us to determine if belt tension has been lost at any point and determine d using the appropriate equation.

That said, since it is very challenging to directly measure both variables on an assembled machine, we must infer their values by working backwards from deflection measurements.

Is it reasonable to assume $F < T$

Manufacturer	
XCarve	Instruction manual suggests 3 pounds force when lifting the belt by 1”.

	Assuming a middle of the road tensile modulus of 30K N/mm/mm and a belt length of 1m this would mean that the belt is tensioned to **92.8N**. https://inventables.zendesk.com/hc/en-us/articles/360012593173-How-do-I-tune-and-calibrate-the-X-Carve-
Shapeoko	Carbide does not seem to offer official figures for belt tension besides the description to tension the belts until they are "guitar string tight". That said, there is a youtube video from 2015 that mentioned 10-15 pounds belt tension (44.5-66.7N), and an extensive forum post that seems to suggest tensioning the belts to between 90-120N. https://www.youtube.com/watch?v=_IIlb_PdziA&t=47s https://community.carbide3d.com/t/measuring-belt-tension-squaring-and-calibration/24712 https://community.carbide3d.com/t/backlash-deflection-and-vibration/28669

If $T > F$, belt tension is always maintained and the standard deflection equation can be used.

$$F = d(T + \lambda)(\frac{1}{x} + \frac{1}{L-x})$$

Since $\lambda >> T$, the equation can be further simplified to the following.

$$F = d\lambda(\frac{1}{x} + \frac{1}{L-x})$$

5. $2T > F$ (e.g. Belts must be pre-tensioned to 50N if load is 100N)
6. Load is applied away from the shorter belt
7. $\lambda >> T$
8. Backlash & hysteresis values are independent of belt length

The first two assumptions allow us to

$$d(T + \lambda)(\frac{1}{x} + \frac{1}{L-x})$$

If $\lambda \gg T$, additional stiffening caused by pre-tensioning can be ignored which further simplifies the equation to:

$$F = d\lambda(\frac{4}{L})$$

Other Implications

Excessive pre-tensioning beyond the amount needed to prevent slack has minimal effects in making the machine “stiffer”. Therefore one should only pre-tension any belts / lead screws to the maximum cutting force the machine is likely to experience.

Table warping

The wasteboard is considered the datum for this test so any warp will be included in the overall deflection measurements. This is a deliberate simplification since we are mostly interested in the deflection of the bit relative to the workpiece (which is secured on the wasteboard).

Key exclusions

Machine behavior in motion is not evaluated in this test.
